

# The Narrow $\Theta_5^+$ Pentaquark As The First Non-planar Hadron With the Diamond Structure And Negative Parity

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(Dated: February 1, 2008)

Using the picture of the flux tube model, we propose that the  $\Theta_5^+$  pentaquark as the first candidate of the three-dimensional non-planar hadron with the extremely stable diamond structure. The up and down quarks lie at the corners of the diamond while the anti-strange quark sits in the center. Various un-excited color flux tubes between the five quarks bind them into a stable and narrow color-singlet. Such a configuration allows the lowest state having the negative parity naturally. The decay of the  $\Theta_5^+$  pentaquark into the nucleon and kaon requires the breakup of the non-planar diamond configuration into two conventional planar hadrons, which involves some kind of structural phase transition as in the condensed matter physics. Hence the width of the  $\Theta_5^+$  pentaquark should be narrow despite that it lies above the kaon nucleon threshold. We suggest that future lattice QCD calculation adopt non-planar interpolating currents to explore the underlying structure of the  $\Theta_5^+$  pentaquark.

PACS numbers: 12.39.Mk, 12.39.-x

Keywords: Pentaquark, Diamond Structure

## I. INTRODUCTION

Up to now nine experimental Collaborations reported evidence for the existence of the narrow  $\Theta^+$  pentaquark state with the minimal quark content  $uudd\bar{s}$  around 1540 MeV [1]. The third component of its isospin is  $I_z = 0$  while its total iso-spin is likely  $I = 0$ . Its angular momentum and parity have not been determined from experiments. Later, NA49 Collaboration [2] reported another narrow pentaquark candidate  $\Xi_5^{--}$  with strangeness  $S = -2$ , baryon number  $B = 1$ , and isospin  $I = \frac{3}{2}$  around 1862 MeV. However the existence of this state is still under debate [3] and confirmation from other groups is necessary.

These exotic baryons are clearly beyond the conventional quark model, which has been successful in the classification of hadron states although its foundation has not been derived from Quantum Chromo-dynamics (QCD). Non-conventional quark model states such as glueballs, hybrid mesons, other multi-quark hadrons are allowed in the QCD spectrum, which have motivated the extensive experimental searches for the past several decades. Unfortunately none of them was pinned down without controversy [4]. The emergence of the  $\Theta^+$  pentaquark finally unveiled one corner of the curtain over the vast landscape of multi-quark hadrons, creating both confusion and anticipation.

Theoretical study of pentaquark states dated back to the early days of QCD using MIT bag model [6]. The re-

cent interest in the  $\Theta^+$  pentaquark started with the prediction of its mass, width and reaction channel from the chiral soliton model (CSM) in Ref. [7] a few years ago. However this prediction is very sensitive to the model inputs [7, 8, 9]. For example, either adopting the commonly used value 45 MeV for the  $\sigma$ -term or identifying  $N(1710)$  as a member of the anti-decuplet will lead to a  $\Xi_5^{--}$  pentaquark mass 210 MeV higher than that observed by NA49 Collaboration [7]. With the new larger value  $(79 \pm 7)$  MeV and  $(64 \pm 7)$  MeV from two recent analysis [10] for the  $\sigma$ -term, a fairly good description of both  $\Theta^+$  and  $\Xi^{--}$  masses is possible [9]. In CSM, the narrow width comes from the unnatural and accidental cancellation between the coupling constants in the leading order, next-leading order and next-next-leading order large  $N_c$  expansion. Moreover, the theoretical foundation of the treatment of the pentaquarks in the chiral soliton model is challenged by the large  $N_c$  formalism in Refs. [11].

Since early last year, there appeared many theoretical papers trying to interpret these exotic states [12, 13, 14, 15, 16, 17, 18, 19, 20, 22, 23, 24, 25, 26]. Several clustered quark models [13, 15, 16] were constructed to ensure the pentaquarks have positive parity as in the original chiral soliton model. But the  $\Theta^+$  pentaquark parity is still a pending issue. For example, one recent lattice calculation favors positive parity for pentaquarks [21] while two previous lattice simulations favor negative parity in Refs. [20]. QCD sum rule approach also favors negative parity [14, 19]. Some other models favor negative parity as well [22]. Recently, many theoretical papers proposed interesting ways to determine  $\Theta^+$  parity [26].

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Among these models, Jaffe and Wilczek's (JW) clustered diquark model is a typical one [13]. They proposed that there exists strong correlation between the light quark pair when they are in the anti-symmetric color ( $\bar{3}_c$ ), flavor ( $\bar{3}_f$ ), isospin ( $I = 0$ ) and spin ( $J = 0$ ) configuration [13]. The lighter the quarks, the stronger the correlation, which helps the light quark pair form a diquark. For example, the  $ud$  diquark behaves like a scalar with positive parity. Such correlation may arise from the color spin force from the one gluon exchange or the flavor spin force induced by the instanton interaction. In order to accommodate the  $\Theta^+$  pentaquark, Jaffe and Wilczek required the flavor wave function of the diquark pair to be symmetric  $\bar{\mathbf{6}}_f$  and their color wave function to be antisymmetric  $\mathbf{3}_c$ . Bose statistics of the scalar diquarks demands an odd orbital excitation between the two diquarks, which ensures that the resulting pentaquark parity is positive. The flavor anti-decuplet is always accompanied by an octet which is nearly degenerate and mixes with the decuplet.

Jaffe and Wilczek pointed out that one of the decay modes  $\Xi_5 \rightarrow \Xi^* + \pi$  observed by NA49 [2] signals the existence of an additional octet around 1862 MeV together the anti-decuplet since the latter can not decay into a decuplet and an octet in the  $SU(3)_f$  symmetry limit. If further confirmed, this experiment poses a serious challenge to the chiral soliton model because there is no baryon pentaquark octet in the rotational band in this model. But it may be hard to exclude it since there always exist excited vibrational octet modes. These modes can not be calculated rigourously within the chiral soliton model [9].

However, in JW's model  $\Theta^+$  is not the lightest pentaquark as in the chiral soliton model. The ideal mixing between the octet and anti-decuplet will split the spectrum and produce two nucleon-like states. The higher one  $N_s$  is around 1710 MeV with a quark content  $qqq\bar{s}s$  where  $q$  is the up or down quark. The lighter one  $N_l$  is lower than the  $\Theta^+$  pentaquark with a quark content  $qqq\bar{q}q$ . Jaffe and Wilczek identified  $N_l$  as the well-known Roper resonance  $N(1440)$  [13], which is a very broad four-star resonance with a width around  $(250 \sim 450)$  MeV. As two members within the same anti-decuplet, it will be very demanding to explain Roper's large decay width and  $\Theta^+$ 's extremely narrow width simultaneously in a natural way. Further analysis of the phenomenological constraints on this model can be found in [27].

## II. THE NARROW WIDTH PUZZLE

According to these experiments, both  $\Theta^+$  and  $\Xi_5^{--}$  are very narrow states. In fact, the  $\Theta^+$  pentaquark is so narrow that most of the experiments can only set an upper bound around 20 MeV. Recent analysis of kaon nucleon scattering data indicates that the width of the  $\Theta^+$  pentaquark is less than several MeV [5], otherwise they should have shown up in these old experiments.

Experience with conventional excited hadrons shows that their widths are around one hundred MeV or even bigger if they lie 100 MeV above threshold and decay through S-wave or P-wave. For comparison, the  $S = -1$  hyperon  $\Lambda(1520)$   $D_{03}$  state is in the same mass region as the  $\Theta^+$  pentaquark. Its angular momentum and parity is  $J^P = \frac{3}{2}^-$ . Its dominant two-body decay is of D-wave with final states  $N\bar{K}$ ,  $\Sigma\pi$ . With a smaller phase space and higher partial wave, the width of  $\Lambda(1520)$  is  $15.6 \pm 1.0$  MeV [4]. In contrast,  $\Theta^+$  lies above 100 MeV  $KN$  threshold and decays through either S-wave or P-wave with a total width less than several MeV, corresponding to negative or positive parity respectively. If these states are further established and confirmed to have such a narrow width, the most challenging issue is to understand their extremely narrow width in a natural way. Is there a mysterious selection rule which is absent from the conventional hadron interaction? This is the topic of the present paper.

Recently there have been several attempts to explain the narrow width of the  $\Theta^+$  pentaquark [23, 24, 25]. Carlson et al. constructed a special pentaquark wave function which is totally symmetric in the flavor-spin part and anti-symmetric in the color-orbital part in Ref. [23]. With this wave function they found that the overlap probability between the pentaquark and the nucleon kaon system is  $\frac{5}{96}$ . Taking into account of the orbital wave function in JW's diquark model further reduces the overlap probability to  $\frac{5}{596}$  [23]. The small overlap probability might be responsible for the narrow width of pentaquarks.

In Ref. [24] Karliner and Lipkin proposed that there might exist two nearly degenerate pentaquarks. Both of them decay into the kaon and nucleon. Hence these two states mix with each other by the loop diagram via the decay modes. Diagonalization of the mass matrix leads to a narrow  $\Theta^+$  pentaquark which almost decouples with the decay mode. The pentaquark with the same quantum number is very broad with a width of around 100 MeV, which has escaped the experimental detection so far.

In Ref. [25] Buccella and Sorba suggested that the four quarks are in the  $L = 1$  state and the anti-quark is in the S-wave state inside the  $\Theta^+$  and  $\Xi_5^{--}$  pentaquarks. When the anti-quark picks up a quark to form a meson, two of the other three quarks remain in the  $SU(6)_{FS}$  totally anti-symmetric state which is orthogonal to that of the  $SU(6)_{FS}$  totally symmetric representation for the nucleon octet. This selection rule is exact in the  $SU(3)_f$  symmetry limit. The narrow widths of the  $\Theta^+$  and  $\Xi_5^{--}$  pentaquarks come from the  $SU(3)_f$  symmetry breaking.

## III. THE DIAMOND STRUCTURE FOR THE NARROW $\Theta_5$ PENTAQUARKS

In this note we propose an alternate scheme to explain the narrow width of the  $\Theta^+$  pentaquark.

For the light quarks, their motion inside the hadrons is

fully relativistic. Talking about the spatial configuration may seem misleading at the first sight. We deal with this issue in the framework of the flux tube model [28].

According to this model, the strong color field between a pair of quark and anti-quark forms a flux tube which confines them. The flux tube is not excited for the ordinary mesons. When it is excited, this system appears as a hybrid meson. Similarly there exist flux tubes between the quark pairs inside the baryon. Recent lattice simulations of static baryon interaction tend to support the flux tube picture [29], although whether the flux tube is of Y-shape or  $\Delta$ -Shape is still under debate.

It's interesting to note that all the conventional mesons and baryons are planar hadrons. At every moment, if we could take a picture of these hadrons, we would find that all the quarks and flux tubes inside the hadrons lie on the same plane.

We propose the narrow  $\Theta_5$  pentaquark as the first candidate of the non-planar hadron with the diamond structure. The strange anti-quark sits in the center while the up and down quarks lie at the corners of the diamond. Four flux tubes between the  $\bar{s}$  and four light (u or d) quarks bind the system into a color singlet.

It is understood that the term "center" or "corner" denotes the relative position which the corresponding quark will occupy with the maximum probability. At every moment, if we could take a real-time picture, we would see a centered diamond structure with the maximum probability.

If the isospin symmetry is exact, the four quarks are exactly on the same footing. For the ground state of this system, none of the four flux tubes is excited. And none of the five quarks is orbitally excited. The diamond structure is exact. So the parity of the lowest state is negative. The isospin is zero.

It's well known that the diamond structure is extremely stable in nature. When the  $\Theta_5$  pentaquark decays into the planar kaon and nucleon, this system undergoes a special structural phase transition, breaking the non-planar flux tubes and forming new planar ones. Hence the decay width of the  $\Theta_5$  pentaquark should be small.

Replacing the  $\bar{s}$  by  $\bar{c}$  and  $\bar{b}$  we will arrive at narrow

heavy pentaquarks with the same diamond structure. Replacing the  $\bar{s}$  by  $\bar{u}$  and  $\bar{d}$  will cause annihilation of the light quark pair, thus change the whole picture. We do not discuss this case here.

For the  $\Xi_5$  with quark content  $uuss\bar{d}$ , the diamond structure is severely distorted because of explicit SU(3) symmetry breaking. Now  $\bar{d}$  sits in the center. Although  $\Xi_5$  is also a narrow state, its width is bigger than that of  $\Theta_5$  since it is relatively easier to break the distorted diamond structure.

We strongly urge that future lattice QCD simulations employ non-local and non-planar interpolating currents to explore the pentaquark structure. If the  $\Theta_5$  pentaquark really possess the diamond structure, it may not couple to the local interpolating current with five quark fields at the same point very strongly because of the topology difference.

We note that Liu et al performed the inherent nodal structure analysis to both the square and the equilateral terahedron based on the pure symmetry consideration [30]. They concluded that the parity of the  $\Theta_5$  pentaquark is positive due to the orbital excitation [30], opposite to what we obtained in the present work.

In short summary, we have proposed the diamond structure to explain the narrow width of the pentaquarks. In the future, the full three-dimensional  $\Theta_5$  pentaquark structure may be explored using the Wigner-type phase space distribution as suggested in Ref. [31].

If future experiments confirm the diamond structure for the  $\Theta_5$  pentaquark, we propose the next interesting candidate of the non-planar hadron is the fullerene-like Quark<sub>60</sub> with sixty valence quarks and baryon number B=20. Such a state is obtained by replacing the carbon atom in the  $C_{60}$  by a valence quark and the corresponding QED valence bonds by the QCD flux tubes. Such a cage-like structure may ensure that this B=20 particle is very stable.

This project was supported by the National Natural Science Foundation of China under Grant 10375003, Ministry of Education of China, FANEDD and SRF for ROCS, SEM. S.L.Z. thanks Prof W.-Y. P. Hwang and COSPA center at National Taiwan University for the warm hospitality.

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